WJEC (Wales) Physics A-level

Topic 4.2: Electrostatic and Gravitational Fields of Force

Notes

Fields

When an object is dropped on Earth, it will begin accelerating towards the Earth's surface. A fridge magnet if held close enough will snap itself to the front of the fridge.

In both these cases an object is clearly experiencing a force without being in contact with another object. **Fields** are said to be the cause of this non-contact force, where objects cause these fields depending on their properties.

Objects with mass cause a gravitational field, whilst objects which hold electric charge produce an electric field. Other objects which have either mass or charge experience a force when placed in these fields.

Gravitational Fields

Intuitively, the force produced by gravitational fields is **always attractive.**

Newton's Law of Gravitation describes how the force between two **point masses** depends on the size of each mass and the distance between them. It is given by the following equation:

$$F = \frac{Gm_1 m_2}{r^2}$$

Here m_1 and m_2 are the size of each mass, and r is the distance between them. This shows that the force is **proportional to the product of the masses** and **inversely proportional to the square of the distance between them.** G is called the gravitational constant, and has a value of $6.67 \times 10^{-11}\ m^3 kg^{-1} s^{-1}$.

This force is experienced by **both masses, towards each other.** In fact, this force is another example of **Newton's Third Law!**

As mentioned, this equation applies only to **point masses.** A point mass is an object which is said to have all of its mass concentrated into a single point, effectively having zero volume. Of course in reality point masses don't exist. However, the gravitational force experienced due to a spherical mass is the same as the force produced by having all of the mass of the sphere located at its center. This means that spherical objects (such as planets) can be thought of as being point masses, allowing us to use Newton's Law of Gravitation for distances outside of the planets radius.

Suppose we took the expression for F and divided both sides by the second mass m_2. This would give:

$$\frac{F}{m_2} = \frac{Gm_1}{r^2}$$

This quantity of the left hand side is be the force experienced **per unit mass** due to the gravitational field produced by m_1. This quantity is called the **gravitational field strength,** which is usually given the symbol g. Also notice that the quantity on the left hand side is also the acceleration of the second mass caused by the force, and for this reason sometimes g is referred to as the **acceleration due to gravity.**

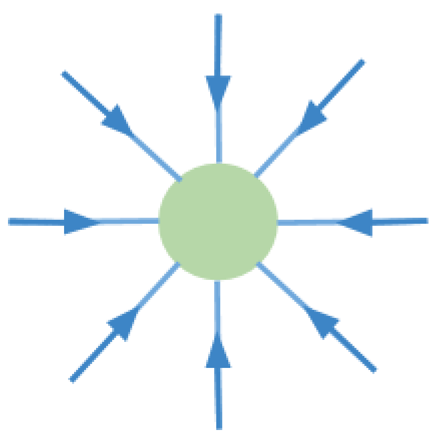
Therefore the gravitational field strength, or acceleration due to gravity produced by a mass M is given by:

$$g = \frac{GM}{r^2}$$

Gravitation Field Lines

The gravitational field strength is a **vector quantity.** This is because it has a size (given by $g = \frac{GM}{r^2}$) and a direction. Seeing as gravitational forces are always attractive, this direction is always towards the centre of the mass M. For this reason, the gravitational field outside of a sphere is described as being **radial.**

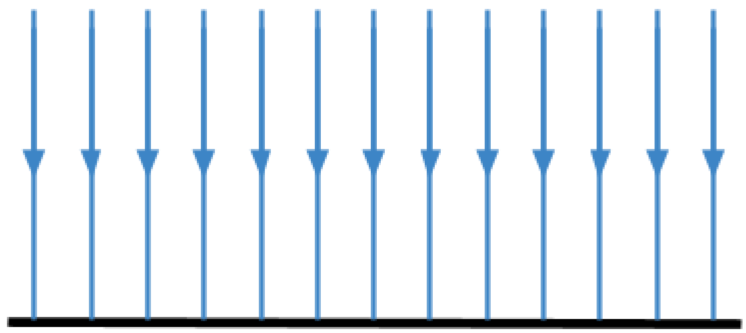
We can illustrate a radial field like so:



Radial field

These blue lines are called **gravitational field lines,** which indicate the direction of force that a mass placed in a field would experience.

If we were to "zoom in" on one small area on the surface of the sphere, the blue field lines would still be radial but the angle between them would become smaller. The more we "zoomed in" the closer these field lines would get to being perfectly parallel. This situation is illustrated here:



Uniform field

In this small region along the surface of the sphere, the field lines are all **approximately parallel.** Objects regardless of where they are placed in this field will experience a force in the same direction. This is what we experience on the surface of Earth: objects always appear to fall downwards. In a uniform field, not only is the direction constant, so is the **size of the field strength.**

Gravitational Potential

Gravitational potential (V) at a point is the **work done per unit mass when moving an object from infinity to that point**. Gravitational potential **at infinity is zero**, and as an object moves from infinity to a point, energy is released as the gravitational potential energy is reduced, therefore **gravitational potential is always negative**.

$$V = -\frac{GM}{r}$$ (For a radial field)

Where M is the mass of the object causing the field, r is the distance between the centres of the objects.

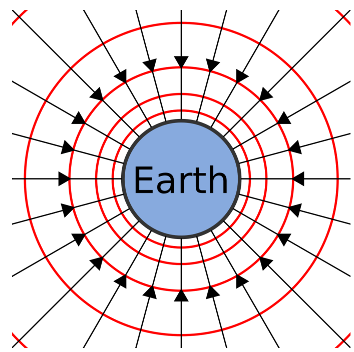
The gravitational potential *energy* is obtained by multiplying the potential by the objects mass. This gives the potential energy for a radial field as:

$$V = -\frac{GMm}{r}$$

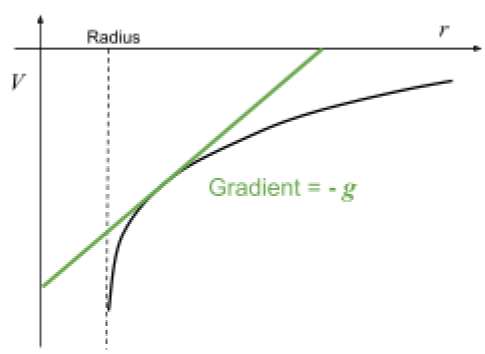
Where m is the mass of the object in the field.
The **gravitational potential difference(ΔV)** is the **energy needed to move a unit mass between two points** and therefore can be used to find the work done when moving an object in a gravitational field.

$$Work\ done = m\Delta V$$ Where m is the mass of the object moved.

Equipotential surfaces are surfaces which are created by joining points of equal potential together, therefore the **potential on an equipotential surface is constant everywhere**. As these points all have equal potential, the gravitational potential difference is zero when moving along the surface, so **no work is done when moving along an equipotential surface**. The red lines on the diagram to the right represent equipotential surfaces.



As shown by the equation for gravitational potential above, **gravitational potential (V) is inversely proportional to the distance between the centres of the two objects (r)**. This can be represented on a **graph of potential (V) against distance *r***:



How does the field strength relate to the potential? The more rapidly the potential is changing, the larger the size of the field strength must be. This is because a large change in potential must correspond to a large gravitational force acting on the object, and hence a larger field strength.

If the distance r is increased, the potential will increase meaning that the field strength must act in the opposite direction to this increase in r.

These two effects occur because the **gravitational field strength is negative gradient of the potential:**

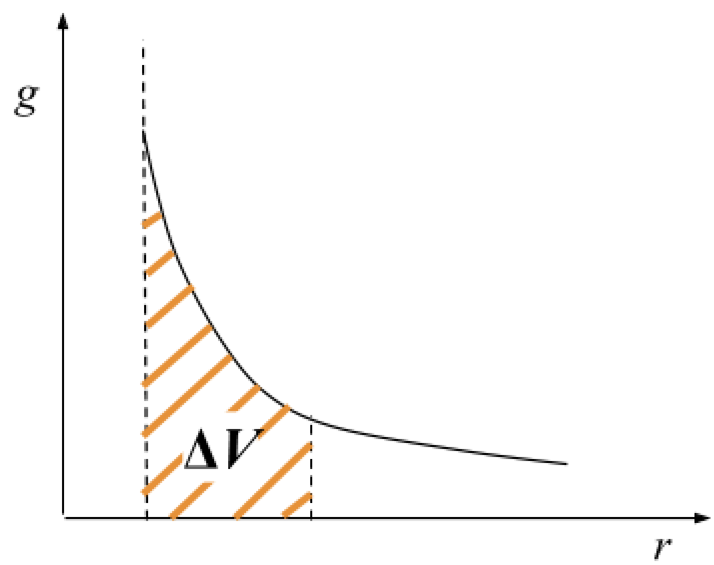
$$g = \frac{-\Delta V}{\Delta r}$$

This means if you know how the potential varies with the radius, then you can draw on a tangent line (as drawn in the diagram above) to find the gradient, then multiply by -1 to obtain the field strength.

Rearranging this equation gives:

$$g\Delta r = -\Delta V$$

This means that if g was constant then the size of the change in potential could be found from the area under the $g-r$ curve. In most cases g isn't constant, but it turns out that the **change in potential is still given by the area under the $g-r$ curve:**



We can make use of the equation $g\Delta r = -\Delta V$ for finding the change in potential energy in situations where g is approximately constant. One instance of this is on the surface of the earth where $g \approx 9.81\ ms^{-2}$. If we move an object through a height h, then we should set $\Delta r = h$. The potential of the object will change by gh. Multiplying by the mass gives the **change in potential energy as:**

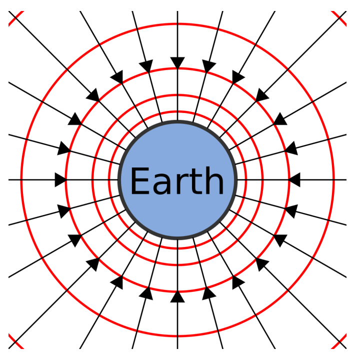
$$\Delta U_p = mgh$$

This equation applies to any situation in which g doesn't change much over the distance h, but is most useful for finding changes in potential energy near Earth's surface where $g \approx 9.81\ ms^{-2}$.

Equipotential Lines

As the name suggests, the **gravitational potential is constant along an equipotential line.** If a mass moves in the direction of a gravitational field, it will do work on the mass, changing its potential energy. This means that if an object moved along a path which was always perpendicular to the gravitational field then no work would be done on it. For this reason, **equipotential lines are always perpendicular to the gravitational field.**

This diagram shows a radial field around Earth:



The red rings drawn are the equipotential lines, seeing as they are always perpendicular to the radial field lines. This means that for a radial field, in 3D the **equipotential surfaces are spheres**.

Electric Fields

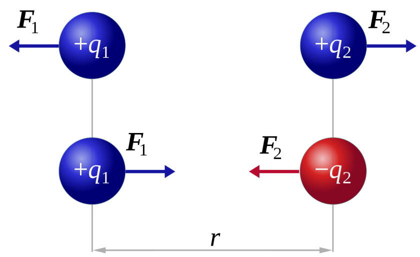
Electric Fields are regions where an **electrically charged** object or particle will experience a force.

Coulomb's Law states that the force experienced between two point charges is **proportional to the product of their charges** and **inversely proportional to the square of the distance** between them. This can be written in equation form as:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2}$$

Where ε_0 is the permittivity of free space, Q_1/Q_2 are charges, r is the distance between charges.

Air can be treated as a vacuum when using the above formula, and for a charged sphere, **charge may be assumed to act at the centre of the sphere**.



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If charges have the **same** sign the force will be **repulsive**, and if the charges have **different** signs the force will be **attractive**.

The **magnitude of electrostatic forces between subatomic particles is magnitudes greater than the magnitude of gravitational forces**, this is because the masses of subatomic particles are incredibly small whereas their charges are much larger.

For example, consider the gravitational and electrostatic force between two protons, with centres 2 pm (2×10^{-12} m) apart.

- **Gravitational force**, $F = \frac{GMm}{r^2}$:

  $$F = \frac{6.67\times10^{-11}\times1.67\times10^{-27}\times1.67\times10^{-27}}{(2\times10^{-12})^2} = 4.65 \times 10^{-41} \text{ N}$$

- **Electrostatic force**, $F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2}$:

  $$F = \frac{1}{4\pi\times8.85\times10^{-12}} \times \frac{1.6\times10^{-19}\times1.6\times10^{-19}}{(2\times10^{-12})^2} = 5.75 \times 10^{-5} \text{ N}$$

- **Ratio of electric force over gravitational force**, $\frac{F_{electric}}{F_{gravitational}}$

  $$\frac{5.75\times10^{-5}}{4.65\times10^{-41}} = 1.24 \times 10^{36}$$

Therefore the electrostatic force between the two protons is 1.24×10^{36} times greater than the gravitational force.

Electric Field Strength

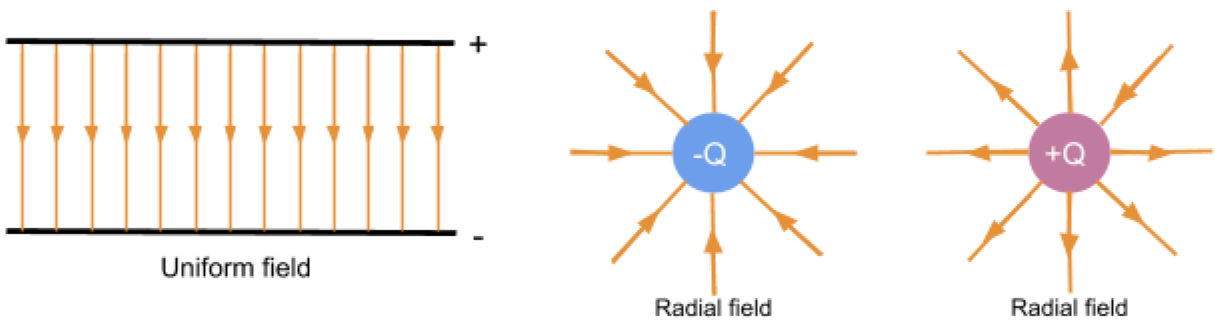
Electric field strength (E) is the force per unit charge experienced by an object in an electric field. This value is constant in a uniform field, but varies in a radial field. There are three formulas you can use to calculate this value; the first is general, the second is used to find the magnitude of E in a uniform field formed by two parallel plates, while the third is used only for radial fields:

$$E = \frac{F}{Q} \qquad E = \frac{V}{d}$$ (for uniform fields formed by parallel plates) $$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$ (for radial fields)

Where V is the potential difference across the plates, d is the distance between the plates, ε_0 is the permittivity of free space, Q is the magnitude of charge, r is the distance between charges.

The value of these constants is approximately $\frac{1}{4\pi\varepsilon_0} \approx 9 \times 10^9\ F^{-1}m$.

Like gravitational fields, electric fields can be uniform or radial and can also be represented by the following field lines:



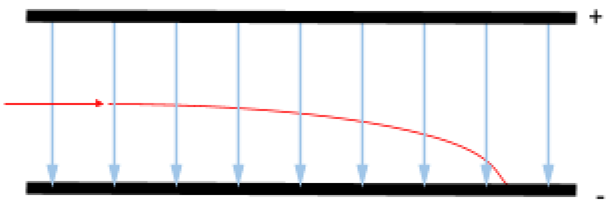
The field lines show the direction of the force acting on a **positive** charge. A **uniform field** exerts that **same** electric force everywhere in the field, as shown by the parallel and equally spaced field lines, whereas in a **radial field** the magnitude of electric force **depends on the distance** between the two charges.

You can derive an equation to calculate the **work done** by **moving a charged particle between the parallel plates** of a uniform field using the equations for electric field strength defined above:

Work done $= F \times d$
Rearrange $E = \frac{F}{Q}$ to get F as the subject. $F = EQ$
Rearrange $E = \frac{\Delta V}{d}$ to get d as the subject. $d = \frac{\Delta V}{E}$
Substitute the above values into the general formula for work done:
Work done = $EQ \times \frac{\Delta V}{E}$ Work done = $Q\Delta V$

Uniform electric fields made by two parallel plates can sometimes be used to find out whether a particle is charged, and whether its charge is negative or positive. This is done by firing the particle at right angles to the field and observing its path: a charged particle will experience a

constant electric force either in or opposite to the direction of the field (depending on its charge), this causes the particle to **accelerate** and so it follows a **parabolic shape**. If the charge on the particle is positive it will follow the direction of the field, if the charge is negative it will move opposite to the direction of the field.



For example, in the diagram to the left, the particle must have a positive charge as it follows a parabolic shape in the direction of the field.

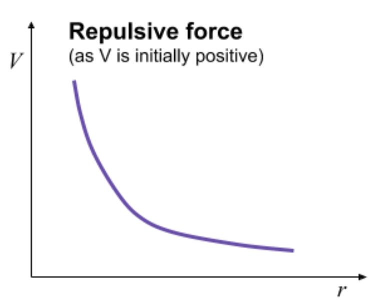
Electric Potential

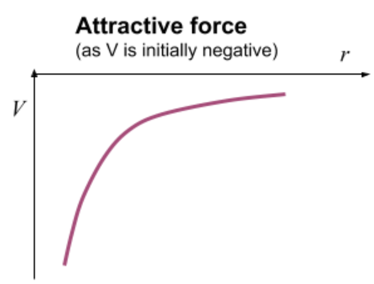
Absolute electric potential (V) at a point is the **potential energy per unit charge of a positive point charge at that point** in the field. The absolute magnitude of electric potential is **greatest at the surface of a charge**, and as the distance from the charge increases, the potential decreases, so **electric potential at infinity is zero**. To find the value of potential in a **radial field** you can use the formula:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

Where ε_0 is the permittivity of free space, Q is the charge, r is the distance from the charge.

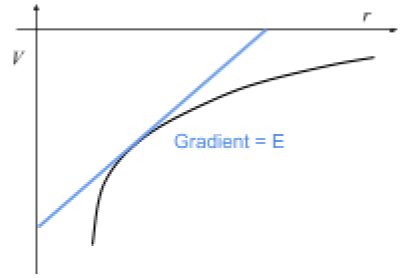
Whether the value of potential is negative or positive depends on the sign of the charge (Q), when the charge is positive, **potential is positive and the charge is repulsive**, when the charge is negative, **potential is negative and the force is attractive**. Note how this is different to gravitational forces which are **always attractive**, this is solely due to the fact that mass is always positive but charge can be negative.





The **gradient of a tangent to a potential (V) against distance (r) graph**, gives the value of **electric field strength (E)** at that point:

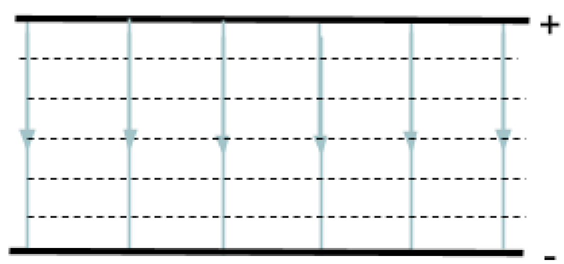
$$E = \frac{\Delta V}{\Delta r}$$

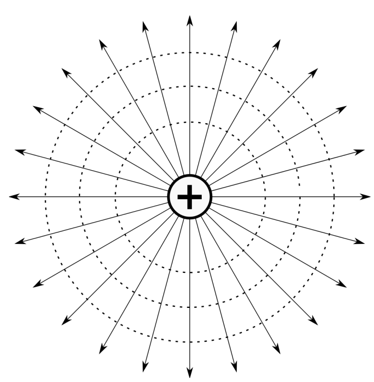


Electric potential difference (ΔV) is the **energy needed to move a unit charge between two points**. Therefore, the work done (ΔW) in moving a charge across a potential difference is equal to the product of potential difference and charge.

$$\Delta W = Q\Delta V$$

Electric fields have **equipotential surfaces**, just like gravitational fields. The potential on an equipotential surface is the same everywhere, therefore **when a charge moves along an equipotential surface, no work is done**. Between two parallel plates the equipotential surfaces are planes which are equally spaced and parallel to the plates, whereas equipotential surfaces around a point charge form concentric circles.





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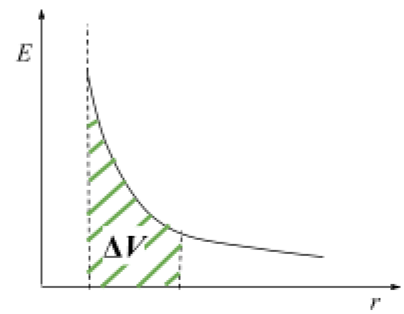
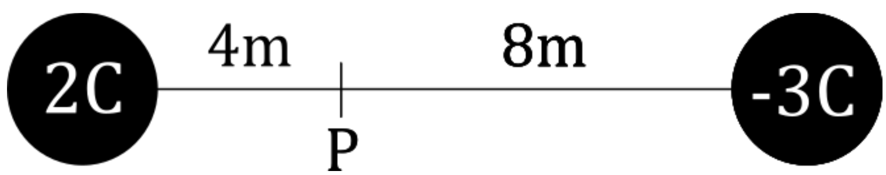
If you plot a graph of **electric field strength (E) against distance (r)**, you can find the **electric potential difference** by finding the **area under the graph**.



Finding the Field and Potential due to Multiple Objects

Both Electric and Gravitational fields satisfy the **superposition principle.** This means that the electric/gravitational field created by multiple objects is equal to the sum of the field produced by each individual object.

For example, what would the electric field be at the point 'P' shown below, between two point charges?



Using the superposition principle, we can find the total electric field by adding the electric fields produced by each charge:

Firstly for the 2C charge, Coulomb's Law gives the electric field as:
$E = \frac{1}{4\pi\varepsilon_0}\frac{2}{4^2} \approx 1.1 \times 10^9\ NC^{-1}$. Then for the second charge, the electric field is:
$E = \frac{1}{4\pi\varepsilon_0}\frac{3}{8^2} \approx 4.2 \times 10^8\ NC^{-1}$. Adding these together gives the total electric field:

$$E_{Total} = 1.1 \times 10^9 + 4.2 \times 10^8 \approx 1.52 \times 10^9\ NC^{-1}.$$

The same idea also applies to the **potential,** or in other words the potential produced by multiple charges is equal to the sum of the potential produced by each individual charge.

In this example we have used electric fields as an example, but the same ideas hold for gravitational fields equally.